Name: ______________________________

Instructor: __________________________

Math 10560, Exam 2
March 21, 2013

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
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8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Total	________

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Multiple Choice

1.(6 pts) Use Simpson's rule with $n = 4$ to estimate

$$\ln 25 = \int_1^5 \frac{5}{x} dx.$$

(a) $\frac{1}{3}\left[\frac{5}{1} + \frac{20}{2} + \frac{20}{3} + \frac{20}{4} + \frac{5}{5}\right]$

(b) $\frac{1}{2}\left[\frac{5}{1} + \frac{10}{2} + \frac{10}{3} + \frac{10}{4} + \frac{5}{5}\right]$

(c) $\frac{2}{3}\left[\frac{5}{1} + \frac{20}{2} + \frac{20}{3} + \frac{20}{4} + \frac{5}{5}\right]$

(d) $\frac{1}{3}\left[\frac{5}{1} + \frac{20}{2} + \frac{10}{3} + \frac{20}{4} + \frac{5}{5}\right]$

(e) $\frac{1}{6}\left[\frac{5}{1} + \frac{20}{2} + \frac{10}{3} + \frac{20}{4} + \frac{5}{5}\right]$

2.(6 pts) Evaluate the improper integral

$$\int_1^4 \frac{1}{(x-2)^3} dx.$$

(a) The integral diverges

(b) 0

(c) $-\frac{7}{16}$

(d) $\frac{7}{16}$

(e) 2

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3.(6 pts) Evaluate the improper integral

$$\int_1^\infty \frac{1}{\sqrt{x}(1+x)}dx.$$

(a) 1

(b) the integral diverges

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

(e) π

4.(6 pts) Which of the following is the correct expression for the arc length of the curve

$$y = e^{\,x^2/2}$$

between the points $(1, \sqrt{e})$ and $(2, e^2)$?

(a) $\int_1^2 \sqrt{1 - x^2 e^{x^2}}\,dx$

(b) $\int_{\sqrt{e}}^{e^2} \sqrt{1 + e^{x^2}}\,dx$

(c) $\int_1^2 \sqrt{1 + x^2 e^{x^2}}\,dx$

(d) $\int_1^2 \sqrt{1 - e^{x^2}}\,dx$

(e) $\int_{\sqrt{e}}^{e^2} \sqrt{1 + x^2 e^{x^2}}\,dx$

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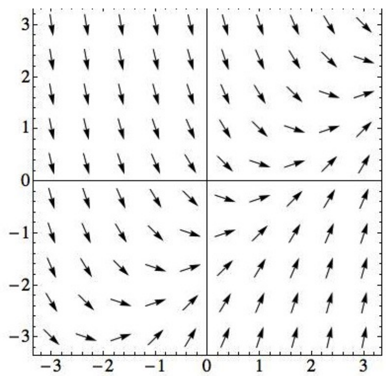
5.(6 pts) Which of the following gives the direction field for the differential equation

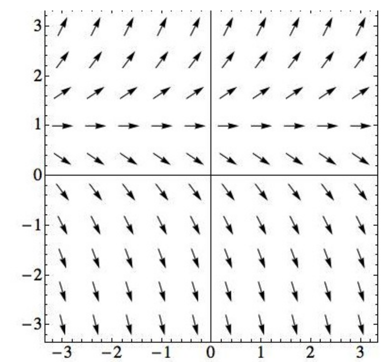
$$\frac{dy}{dx} = y^2 - 1\ ?$$

Note the letter corresponding to each graph is at the **lower left** of the graph.

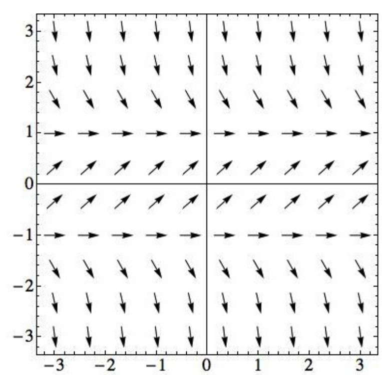
(a)



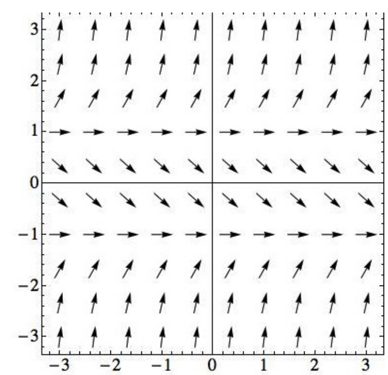
(b)



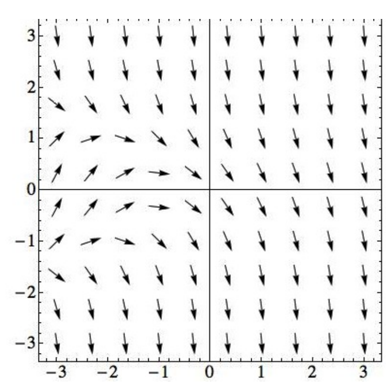
(c)



(d)



(e)



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6.(6 pts) Use Euler's method with step size 0.5 to estimate $y(1.5)$ where $y(x)$ is the solution to the initial value problem

$$y' = y^2 + 2x, \quad y(0.5) = 1.$$

(a) 5 (b) 6 (c) 2 (d) 1 (e) 8.5

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7.(6 pts) Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{3y}{2x+1}.$$

(a) $3y^2 = \frac{1}{2}\ln|2x+1| + C$

(b) $y = \frac{3}{2}\ln|2x+1| + C$

(c) $y = C(2x+1)^{3/2}$

(d) $y = \frac{C}{2}\ln|2x+1|$

(e) $\frac{3y^2}{2} = x^2 + x + C$

8.(6 pts) Find the solution of the differential equation

$$\frac{dy}{dx} - \left[\frac{2x}{x^2+4}\right]y = (x^2+4)\cos x$$

with initial condition $y(0) = 1$.

(a) $y = 1 + \sin x$

(b) $y = \frac{(x^2+4)\cos x}{2x+1}$

(c) $y = \frac{\cos x + 3}{x^2+4}$

(d) $y = \frac{(x^2+4)\sin x}{2x+1}$

(e) $y = (x^2+4)\left(\frac{1}{4} + \sin x\right)$

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9.(6 pts) Determine if the sequence given by $a_n = ne^{-2n}$ converges or diverges and if it converges find

$$\lim_{n\to\infty} ne^{-2n}$$

(a) Converges and $\lim_{n\to\infty} a_n = 1$

(b) Converges and $\lim_{n\to\infty} a_n = 0$

(c) The sequence diverges.

(d) Converges and $\lim_{n\to\infty} a_n = \frac{1}{2}$

(e) Converges and $\lim_{n\to\infty} a_n = 2$

10.(6 pts) Consider the following sequences:

$$(I)\ \left\{(-1)^n \frac{n^2-1}{2n^2+1}\right\}_{n=1}^{\infty} \qquad (II)\ \left\{(-1)^n \frac{n^2-1}{e^n}\right\}_{n=1}^{\infty} \qquad (III)\ \left\{(-1)^n n\ln(n)\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) Sequences I and II converge and sequence III diverges

(b) All three sequences converge.

(c) Sequences II and III converge and sequence I diverges.

(d) All three sequences diverge.

(e) Sequence II converges and sequences I and III diverge.

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (13 pts.) Complete the following sentences using the words *converges* and *diverges* :

$\int_1^\infty \frac{1}{x^p}$ ________________ if $p > 1$ and ________________ if $p \leq 1$.

$\int_0^1 \frac{1}{x^p}$ ________________ if $p > 1$ and ________________ if $p \leq 1$.

Decide whether the following improper integrals converge or diverge by comparing them to a known integral. In each case, state which integral you are comparing the given integral to and state clearly why you can conclude convergence or divergence.

(a) $\displaystyle\int_1^\infty \frac{1}{x^2 + x + 5}\, dx$

(b) $\displaystyle\int_1^\infty \frac{1}{xe^x}\, dx$

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12. (14 pts.) Find the centroid of the region enclosed by the curves $y = x^2$ and $y = x^3$.

$\bar{x} =$_____ $\bar{y} =$ _____

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13. (13 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = k\sqrt{x}.$$

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

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